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A Partially Backlogging Inventory Model for Deteriorating Items with Stock Dependent Selling Rate

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Abstract—*In the present paper we considered an inventory model for deteriorating items with stock dependent selling rate. Shortages are allowed and completely backlogged. The backlogging rate of unsatisfied demand is assumed to be a decreasing exponential function of waiting time. The purpose of our study is to maximize the total profit. Further a numerical example is also given to demonstrate the developed model and to show the sensitivity analysis of the effect of change of parameters.*

Keywords: *Inventory, deterioration, Shortage, Stock dependent demand and partial backlogging.*

1. INTRODUCTION

In some consumer goods it has been observed that the demand of some items is influenced by the amount of on hand inventory stock. The demand of such goods may be increase or decrease according as on hand inventory stock increases or decreases. This situation arise in a super market where is a lot of stock of goods. The effect of deterioration cannot be avoided in any business organization so deterioration is defined by a process in which the stocked items loose part of their value with passage of time.

In past few years some researchers considered the time dependent demand rate because the demand of newly launched products such as fashionable garments, electronic items, mobile phones increases with time and later it becomes constant. In some real life situations the customers suffer the problem of shortage because there are some customers who wait for the next replenishment while the others do not wait for replenishment and go elsewhere as waiting time increases, but in the recent years some researchers gave their attention towards the stock dependent demand rate. Levin et al. [1972] show in his inventory model that in a super market a large pile of goods displayed will attract the customers to buy more. Silver and Peterson [1985] show that the sales at the retail level proportional to the amount of on hand inventory. Padmanabhan and Vrat [1995] developed EOQ models with and without partial backlogging for perishable products with stock dependent demand rate and zero lead time under replenishment. Hou [2005] extend the Padmanabhan and Vrat [1995] partial backlogging inventory model without considering the lost sales due to opportunity cost. Dye and Ouyang [2005] considered an inventory model and show that the proportion of the customers who wait for backlogging is a reciprocal linear function of waiting time.

Abad [1996, 2001] developed a pricing and lot sizing inventory model with a variable rate of deterioration by allowing shortages which are partially backlogged and the backlogging rate depends on the time of replenishment so more customers may queue up for orderand greater the fraction of sales are lost. Chern et al. [2008] developed a partial backlogging and lot sizing inventory model for deteriorating items with inflation and fluctuating demand. The researchers Chang and Dye [1999], Papachristos and Skouri [2003], Wu et al. [2006], Min and Zhou [2009] and Hsies et al. [2010] modified the inventory policies by considering time proportional backlogging rate. Dutta and Pal [1990], Pl et al [1993], Padmanabhan and Vrat [1995], Giri [1996], Sarker et al. [1997], Giri and Chaudhari [1998], Blkhi and Benkherouf [2004], Pal et al. [2006], Hsieh and Dye [2010] and Changet al. [2010] developed the order level lot sizing inventory models for discussing the effects of deterioration and stock dependent demand demand. Ghare and Schrader [1963], Shah [1977], Heng et al. [1991], Wee [1995], Goyal and Giri [2001], Teng et al. [2002] and Sana [2010] developed their inventory models and discussing the issues related to deterioration, effect of inflation and time value of money on ordering policy over a finite time horizon. Buzacott [1975] developed an EOQ model with inflation and subject to the different cases of pricing policies. Hou and Lin [2006] considered an inventory model for deteriorating items with stock and price dependent selling rate. Ray and Chaudhari [1997], Cheng [1998], Sarker et al. [2010], Chung and Lin [2001], Wee and Law [2001], Balkhi [2004], Jaggi et al. [2006] developed their models for deteriorating items under inflation and time value of money.

In the present paper we consider an inventory model for deteriorating items with stock dependent demand rate. Shortages are allowed and completely backlogged. The backlogging rate of unsatisfied demand is assumed as a decreasing exponential function of waiting time. The purpose of our study is to maximize the total profit. A numerical example is also given to demonstrate the developed model and to shoe the sensitivity analysis of the parameters.

2. ASSUMPTIONS AND NOTATIONS

We consider the following assumptions and notations

1. The demand rate is $R(t)=\alpha+\beta I(t)$, where α is a positive constant and β is the stock dependent selling rate parameter, $0\le\beta\le 1$
2. θ is the deterioration parameter.
3. δ is the backlogging parameter.
4. A is the ordering cost per order.
5. h_C is the holding cost per unit per unit time.
6. s_C is the shortages cost per unit per unit time.
7. C is the purchase cost per unit.
8. p is the selling price per unit, where.
9. C_3 is the opportunity cost per unit due to lost sales.
10. T is the length of order cycle.
11. T_1, is the time at which shortage starts.
12. $TP(T_1,T)$ is the total profit per unit time.
13. I(t) is the inventory level at any time in [0,T].
14. The time horizon T is infinite.
15. The lead time is zero.
16. The replenishment rate is infinite.

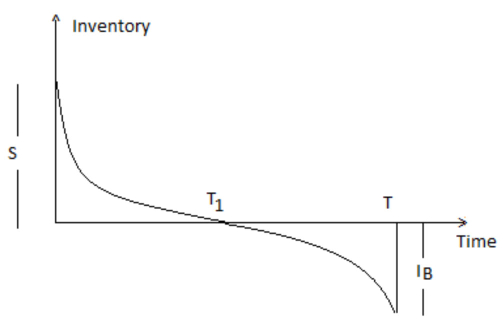

Fig. I

3. MATHEMATICAL FORMULATION-

Suppose an inventory system consists S units of the product in the beginning of each cycle. Due to demand and deterioration the inventory level decreases in $[0,T_1]$ and becomes zero at $t=T_1$. The interval $[T_1,T]$ is the shortages interval. During the shortages interval $[T_1,T]$ the unsatisfied demand is backlogged at a rate of $e^{-\delta t}$, where δ is the backlogging parameter and t is the waiting time.

The instantaneous inventory level at any time t in [0, T] are governed by the following differential equations

$$\frac{dI}{dt}+\theta I=-(\alpha+\beta I),\qquad 0\le t\le T_1 \tag{1}$$

with boundary condition I(0) = S

$$\frac{dI}{dt}=-\alpha e^{-\delta t},\qquad T_1\le t\le T \tag{2}$$

with boundary condition $I(T_1)=0$

The solution of equation (1) is

$$I=\frac{\alpha}{(\theta+\beta)}[e^{(\theta+\beta)t}-1]+S e^{(\theta+\beta)t}$$

$$I=S+\{\alpha+S(\theta+\beta)\}t+(\theta+\beta)\{\alpha+S(\theta+\beta)\}\frac{t^2}{2} \tag{3}$$

The solution of equation (2) is

$$I=\frac{\alpha}{\delta}[e^{-\delta(T-T_1)}-e^{-\delta(T-t)}]$$

$$I=\alpha(T_1-t)+\frac{\alpha\delta}{2}(T_1^2-t^2-2TT_1+2Tt)\ldots \tag{4}$$

The ordering cost per cycle is

$$O_C=\frac{A}{T} \tag{5}$$

The holding cost per cycle is

$$H_C=\frac{h_C}{T}\int_0^{T_1} I(t)\,dt$$

$$H_C=\frac{h_C}{T}\int_0^{T_1}[S+\{\alpha+S(\theta+\beta)\}t+(\theta+\beta)\{\alpha+S(\theta+\beta)\}\frac{t^2}{2}]$$

$$H_C=\frac{h_C}{T}[ST_1+\{\alpha+S(\theta+\beta)\}\frac{T_1^2}{2}+(\theta+\beta)\{\alpha+S(\theta+\beta)\}\frac{T_1^3}{6}] \tag{6}$$

The shortage cost per cycle is

$$S_C=\frac{s_C}{T}\int_{T_1}^{T}-I(t)dt$$

$$S_C=-\frac{s_C}{T}[\alpha\delta(\frac{T^3}{3}-\frac{T_1^3}{3}+TT_1^2-T_1T^2)+\alpha TT_1-\frac{\alpha T^2}{2}-\frac{\alpha T_1^2}{2}] \tag{7}$$

Due to lost sales the opportunity cost per cycle in $[T_1,T]$ is

$$O_{PC} = \frac{\alpha C_3}{T}\int_{T_1}^{T}[1 - e^{-\delta(T-t)}]dt$$

$$O_{PC} = \frac{\alpha\delta C_3}{2}[T^2 + T_1^2 - 2TT_1 + \frac{\delta}{3}(T^3 - T_1^3) - 3T_1T^2 + 3TT_1^2)] \quad (8)$$

The purchase cost per cycle in $[T_1, T]$ is

$$P_C = CS + C \times back\ ordered\ quantity$$

$$P_C = C[S + \alpha(T_1 - T) + \frac{\alpha\delta}{2}(T_1^2 - T^2 - 2TT_1 + 2T^2] \quad . (9)$$

The sales revenue per cycle is

$$S_R = p[\int_0^{T_1}(demand\ )\,dt + \int_{T_1}^{T}(demand\ )dt$$

$$S_R = p\beta[ST_1 + \{\alpha + S(\theta+\beta)\}\frac{T_1^2}{2} + (\theta+\beta)\{\alpha + S(\theta+\beta)\}\frac{T_1^3}{6}] + p\alpha T - \frac{p\alpha\delta}{2}(T - T_1)^2 + \frac{p\alpha\delta^2}{6}(T - T_1)^3 \quad (10)$$

Therefore the total profit per unit time is

$$TP(T_1, T) = \frac{1}{T}[S_R - O_C - H_C - S_C - O_{PC} - PC]$$

$$TP(T_1, T) = \frac{1}{T}[-(A + CS) + (p + C)\alpha T + \{p\beta S - Sh_C - \alpha C\}T_1 + \{\alpha s_C - \alpha\delta p - \alpha\delta C_3 - \alpha\delta C\}\frac{T^2}{2} + \{p\beta(\alpha + S(\theta+\beta)) - \alpha\delta p + \alpha s_C - h_C(\alpha + S(\theta+\beta)) - \alpha\delta C_3 - \alpha\delta C\}\frac{T_1^2}{2} + \{\alpha\delta p - \alpha s_C + \alpha\delta C_3 + \alpha\delta C\}TT_1 + \{\alpha\delta^2 p - 2\alpha\delta s_C - \alpha\delta^2 C_3\}\frac{T^3}{6} + \{p\beta(\theta+\beta)(\alpha + S(\theta+\beta)) - \alpha p\delta^2 + 2\alpha\delta s_C - h_C(\theta+\beta)(\alpha + S(\theta+\beta)) + \alpha\delta^2 C_3\}\frac{T_1^3}{6} + \{2\alpha\delta s_C - \alpha p\delta^2 + \alpha C_3\delta^2\}\frac{T_1T^2}{2} + \{\alpha p\delta^2 - 2\alpha\delta s_C - \alpha C_3\delta^2\}\frac{TT_1^2}{2}] \quad \ldots \quad ..... (11)$$

The necessary condition for $TP(T_1, T)$ to be maximum is that $\frac{\partial TP(T_1,T)}{\partial T_1} = 0$ *and* $\frac{\partial TP(T_1,T)}{\partial T} = 0$, and solving these equations we find the optimum values of T_1 *and* T say T_1^* *and* T^* for which profit is maximum and the sufficient condition is

$$(\frac{\partial^2 TP(T_1,T)}{\partial T_1^2})(\frac{\partial^2 TP(T_1,T)}{\partial T^2}) - \{\frac{\partial^2 TP(T_1,T)}{\partial T_1 \partial T}\}^2 \succ 0$$

$$and\ (\frac{\partial^2 TP(T_1,T)}{\partial T_1^2}) \prec 0 \quad \ldots\ldots\ldots \quad (12)$$

$$-h_C(\alpha + S(\theta+\beta))\}T_1 + \{\alpha\delta p - \alpha s_C + \alpha\delta C + \alpha\delta C_3\}T + \{p\beta(\theta+\beta)(\alpha + S(\theta+\beta)) - \alpha p\delta^2 + 2\alpha\delta s_C - h_C(\theta+\beta)(\alpha + S(\theta+\beta)) + \alpha C_3\delta^2\}\frac{T_1^2}{2} + \{2\alpha\delta s_C - \alpha p\delta^2 + \alpha C_3\delta^2\}\frac{T^2}{2} + \{\alpha p\delta^2 - 2\alpha\delta s_C - \alpha C_3\delta^2\}TT_1] \ \ldots\ldots \ . (13)$$

$$\frac{\partial^2 TP(T_1,T)}{\partial T_1^2} = \frac{1}{T}[\{p\beta(\alpha + S(\theta+\beta)) - \alpha\delta p + \alpha s_C - \alpha\delta C - \alpha\delta C_3 - h_C(\alpha + S(\theta+\beta))\} + \{p\beta(\theta+\beta)(\alpha + S(\theta+\beta)) - \alpha p\delta^2 + 2\alpha\delta s_C - h_C(\theta+\beta)(\alpha + S(\theta+\beta)) + \alpha C_3\delta^2\}T_1 + \{\alpha p\delta^2 - 2\alpha\delta s_C - \alpha C_3\delta^2\}T] \quad (14)$$

$$\frac{\partial TP}{\partial T} = \frac{1}{T}[(p + C)\alpha + \{\alpha s_C - \alpha\delta p - \alpha\delta C_3 - \alpha C\delta\}T + \{\alpha p\delta - \alpha s_C + \alpha\delta C_3 + \alpha C\delta\}T_1 + \{\alpha p\delta^2 - 2\alpha\delta s_C - \alpha\delta^2 C_3\}\frac{T^2}{2} + \{2\alpha\delta s_C - \alpha p\delta^2 + \alpha\delta^2 C_3\}TT_1 + \{\alpha p\delta^2 - 2\alpha\delta s_C - \alpha\delta^2 C_3\}\frac{T_1^2}{2}] - \frac{1}{T^2}[-(A + CS) + (p + C)\alpha T + \{p\beta S - Sh_C - \alpha C\}T_1 + \{\alpha s_c - \alpha\delta p - \alpha\delta C_3 - \alpha C\delta\}\frac{T^2}{2} + \{p\beta(\alpha + S(\theta+\beta))$$

$$-\alpha\delta p+\alpha s_C-\alpha\delta C_3-\alpha\delta C-h_C(\alpha+S(\theta+\beta))\}\frac{T_1^2}{2}+\{\alpha p\delta-\alpha s_c+\alpha\delta c_3+\alpha C\delta\}TT_1+\{\alpha p\delta^2-2\alpha\delta s_C-\alpha\delta^2C_3\}\frac{T^3}{6}+\{p\beta(\theta+\beta)(\alpha+S(\theta+\beta))-\alpha p\delta^2+2\alpha\delta s_C-h_C(\theta+\beta)(\alpha+S(\theta+\beta))-\alpha\delta^2C_3\}\frac{T_1^3}{6}+\{2\alpha\delta s_C-\alpha p\delta^2+\alpha\delta^2C_3\}\frac{T_1T^2}{2}+\{\alpha p\delta^2-2\alpha\delta s_C-\alpha\delta^2C_3\}\frac{TT_1^2}{2}] \quad (15)$$

$$\frac{\partial^2 TP}{\partial T^2}=\frac{1}{T}[\{\alpha s_C-\alpha p\delta-\alpha\delta c_3-\alpha C\delta\}+\{\alpha p\delta^2-2\alpha\delta s_C-\alpha\delta^2C_3\}T+\{2\alpha\delta s_C-\alpha p\delta^2+\alpha\delta^2C_3\}T_1]-\frac{1}{T^2}[(p+C)\alpha+\{\alpha s_C-\alpha p\delta-\alpha\delta c_3-\alpha C\delta\}T+\{\alpha\delta p-\alpha s_c+\alpha\delta c_3+\alpha C\delta\}T_1+\{\alpha p\delta^2-2\alpha\delta s_C-\alpha\delta^2C_3\}\frac{T^2}{2}+\{2\alpha\delta s_C-\alpha p\delta^2+\alpha\delta^2C_3\}TT_1+\{\alpha p\delta^2-2\alpha\delta s_C-\alpha\delta^2C_3\}\frac{T_1^2}{2}]+\frac{2}{T^3}[-(A+CS)+(p+C)\alpha T+\{p\beta S-Sh_C-\alpha C\}T+\{\alpha s_C-\alpha p\delta-\alpha\delta c_3-\alpha C\delta\}\frac{T^2}{2}+\{p\beta(\alpha+S(\theta+\beta))-\alpha\delta p+\alpha s_C-\alpha\delta C_3-\alpha\delta C-h_C(\alpha+S(\theta+\beta))\}\frac{T_1^2}{2}+\{p\beta(\theta+\beta)(\alpha+S(\theta+\beta))-\alpha p\delta^2+2\alpha\delta s_C-h_C(\theta+\beta)(\alpha+S(\theta+\beta))-\alpha\delta^2C_3\}\frac{T_1^3}{6}+\{2\alpha\delta s_C-\alpha p\delta^2+\alpha\delta^2C_3\}\frac{T_1T^2}{2}+\{\alpha p\delta^2-2\alpha\delta s_C-\alpha\delta^2C_3\}\frac{TT_1^2}{2}] \quad (16)$$

$$\frac{\partial^2 TP}{\partial T\partial T_1}=\frac{1}{T}[\{\alpha\delta p-\alpha s_C+\alpha\delta c_3+\alpha C\delta\}+\{2\alpha\delta s_C-\alpha p\delta^2+\alpha\delta^2C_3\}T+\{\alpha p\delta^2-2\alpha\delta s_C-\alpha\delta^2C_3\}T_1]-\frac{1}{T^2}[(p\beta S-Sh_C-\alpha C)+\{p\beta(\alpha+S(\theta+\beta))-\alpha\delta p+\alpha s_C-\alpha\delta C_3-\alpha\delta C-h_C(\alpha+S(\theta+\beta))\}T_1+\{\alpha p\delta-\alpha s_c+\alpha\delta c_3+\alpha C\delta\}T+\{p\beta(\theta+\beta)(\alpha+S(\theta+\beta))-\alpha p\delta^2+2\alpha\delta s_C-h_C(\theta+\beta)(\alpha+S(\theta+\beta))-\alpha\delta^2C_3\}\frac{T_1^2}{2}+\{2\alpha\delta s_C-\alpha p\delta^2+\alpha\delta^2C_3\}\frac{T^2}{2}+\{\alpha p\delta^2-2\alpha\delta s_C-\alpha\delta^2C_3\}TT_1] \quad (17)$$

Numerical example-Let us consider the following parameters in the appropriate units
$\alpha=200,\ \beta=0.5,\ \theta=0.005,\ \delta=2,\ A=100,$
$h_C=3,\ s_C=4,\ C_3=5,\ C=6,\ p=10,\ S=500$

Table I

θ	T_1	T	TP
0.005	36.3938	818.2250	7.5046×10^7
0.010	36.3942	818.3942	7.5050×10^7
0.015	36.3945	818.2710	7.5055×10^7
0.020	36.3949	818.2940	7.5059×10^7

As we increase the parameter θ then the values of the parameters, T_1, T and TP increases.

Table II

β	T_1	T	TP
0.5	36.3938	818.2250	7.5046×10^7
1.0	36.6280	832.8690	8.1140×10^7

2.0	38.1901	938.5680	1.0084×10^8
3.0	41.1326	1174.5800	1.6250×10^8

As we increase the parameter β then the values of the parameters, T_1, T and TP increases.

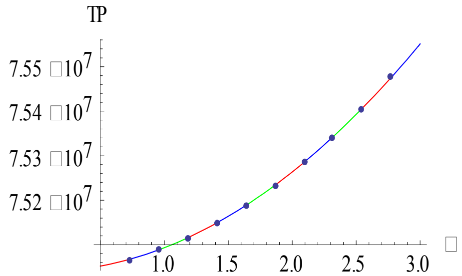

Fig. II

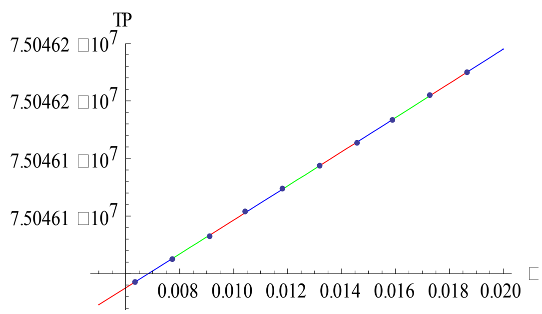

Fig. III

4. CONCLUSION

In this paper we considered an inventory model for deteriorating items with stock dependent selling rate. Shortages are allowed and completely backlogged. The backlogging rate of unsatisfied demand is assumed to be a decreasing exponential function of waiting time. From the tables I and II we see that as we increase the deterioration and selling rate parameters θ and β the cycle time, time at which shortages start and total profit increases. The parameter β is more sensitive than the parameter θ. In future this model can be generalized by considering time dependent deterioration, holding and shortage cost.

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